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STUDY OF THE RHEOLOGICAL PROPERTIES OF CLAYS USING THE PROBLEM ON SHRINKAGE OF A CLAY LAYER AS AN EXAMPLE

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The rheological properties of water-saturated clays have been studied based on the model described in [1, 2]. The problem on shrinkage of a clay layer under strain has been reduced to the well-known problem of N. N. Verigin. The numerical solution of the problem on squeezing of water from a clay layer has been found and analyzed. The distinctive features of the model, which are important for explanation of certain characteristic features of the rheology of clays, have been investigated. It has been shown that the solutions obtained are in qualitative agreement with experimental results.

Introduction. Study of the rheological properties of clays is traditionally of interest from both the applied viewpoint (clays are a raw material for manufacture of drilling muds, ceramics, and brick) and the theoretical viewpoint. In the latter case, the problem is reduced, as a rule, to the following one: to put forward a system of hypotheses for the structure and properties of a medium and processes occurring in it such that none of them is inconsistent with the available experimental facts and to formulate, based on it, a mathematical model which could be realized for a standard set of known phenomenological constants for clays and processes in them and would lead to numerical results consistent with experiment. The difficulties of mathematical modeling of the rheological properties of clays are associated with the necessity of allowing for the processes of mechanical straining of a medium and changes in the packing of clay particles and the distinctive features of the motion of water in macro- and micropores of a clay rock. These problems have been studied in many works, including [1]. In it, it was, in particular, proposed that the problem obtained for the simplest case of one-dimensional shrinkage of a clay layer under a constant load be reduced to the Verigin problem [2]. Physically this corresponds to the appearance of two shrinkage zones in the clay layer, in one of which the pores of the clay rock are only partially filled with clay particles and consequently the permeability of this zone is relatively high, whereas in the other, clay particles entirely fill the pores of the rock and the permeability of this zone is lower than that of the first zone. Thus, any process associated with the straining of clay rocks can be subdivided into two stages, in which (1) the fraction of pores free from clay particles (free porosity) is not equal to zero and (2) free porosity is equal to zero, i.e., clay particles have entirely filled all the pores.

In the simplest case, in the first stage, the process of shrinkage of a porous medium with an elastic skeleton due to the squeezing of a fluid from the pores of the rock [3] is described by the piezoconductivity equation [4]

$$\frac{\partial p_1}{\partial t} = \chi_1 \nabla^2 p_1 \,. \tag{1}$$

The elastic properties of the skeleton are characterized by the constant λ; the piezoconductivity coefficient χ is related to the parameters of the medium and the fluid by the relation $\chi = k\lambda/\eta$.

In the second stage, all the pores of the clay rock are clogged with clay particles, and water flows between them. In this case we can also apply Eq. (1) but with a smaller piezoconductivity coefficient χ_2, since the resistance to flow is higher in this case and accordingly the permeability of the medium is lower:

$$\frac{\partial p_2}{\partial t} = \chi_2 \nabla^2 p_2 \,. \tag{2}$$

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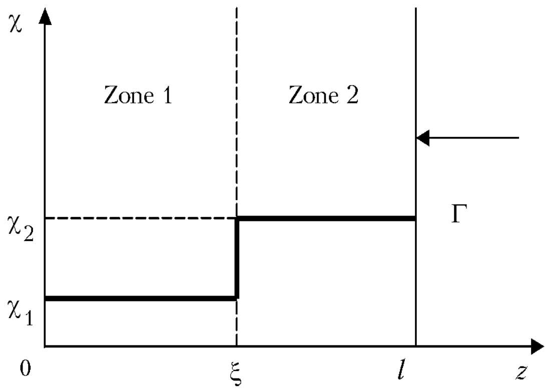


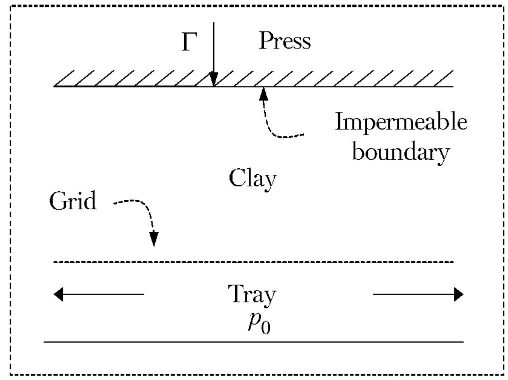


Fig. 1. Formation of shrinkage zones with different piezoconductivity values and a moving boundary between the zones.

Fig. 2. Diagram of the experiment on straining of the clay layer under a constant external load Γ.

Consequently, the piezoconductivity coefficient χ at the boundary between the zones formed changes abruptly, and we obtain the so-called Verigin problem [5]

$$\frac{\partial p_1}{\partial t} = \chi_1 \frac{\partial^2 p_1}{\partial z^2}, \quad 0 \le z < \xi; \qquad \frac{\partial p_2}{\partial t} = \chi_2 \frac{\partial^2 p_2}{\partial z^2}, \quad \xi < z \le l, \tag{3}$$

where ξ is the moving boundary of contact of two zones (Fig. 1). The approximate analytical solution of the problem leads to the Bingham rheological problem with a limit shear stress [2]. Below we give the numerical solution of problem (3).

Formulation of the Problem. Let us visualize a clay layer in which the process of squeezing of water occurs. A constant external load acts on the layer from above, due to which the water in the pore (void) space flows out into a tray where the pressure is constant and equal to p_0 (Fig. 2). Such a diagram simulates a real experiment. As has already been mentioned above, processes occurring in the layer are described by Eqs. (3). Now it is necessary to allow for the condition of impermeability of the layer's upper boundary:

$$\frac{\partial p_2}{\partial z} = 0\,. \tag{4}$$

Let us consider problem (3) with boundary condition (4) in the domain D:$\{z = (0, l), t = (0, T)\}$. It is known that the condition of constancy of the flow

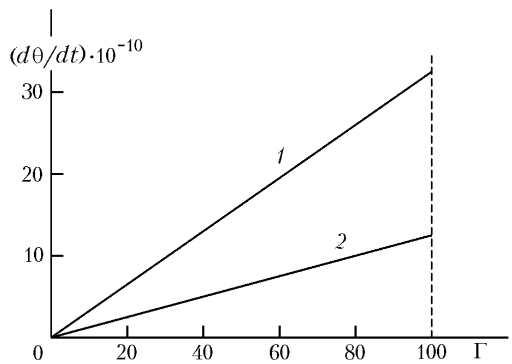
$$\chi_1 \frac{\partial p}{\partial z}(\xi - 0, t) = \chi_2 \frac{\partial p}{\partial z}(\xi + 0, t) \tag{5}$$

holds at the boundary of two zones (where χ_1 is abruptly replaced by χ_2). The initial conditions are as follows:

$$p(z, 0) = (p_0 - \Gamma)(1 - \exp(z)) + \Gamma\,,$$
$$p(0, t)\big|_{t=0} = \Gamma\,, \quad \xi(0) = 0\,, \quad \xi(T) = l\,, \quad p_0 = 0\,. \tag{6}$$

For computation of the shrinkage θ we know the dependence [3, 6]

$$\Gamma = \lambda\theta + p(z, t)\,, \tag{7}$$



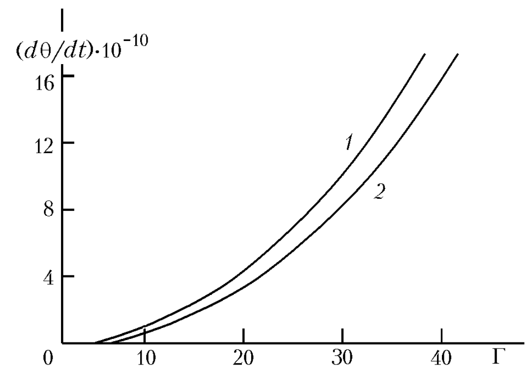


Fig. 3. Mean shrinkage rate vs. load at different averaging times: 1) 40 and 2) 100 sec. Γ, kPa.

Fig. 4. Mean shrinkage rate vs. external load: 1) $\chi_1 = 0.3$ and $\chi_2 = 0.03$; 2) 0.7 and 0.07. Γ, kPa.

where (for evaluation purposes) it is taken that $\lambda = E \approx 10^{10}$ N/m^2. Thus, knowing the pressure at any point of the bed at any instant of time, we can calculate the shrinkage and the derivatives of it. The problem obtained requires that the constant λ of the skeleton material directly, i.e., a dry clay rock, be experimentally determined in each particular case. Therefore, it is necessary to obtain the calculated dependences based on the above evaluation by λ. Thus, the calculation results are model in character.

Calculations and Discussion of the Results. First of all, we investigated changes in the mean value of the shrinkage rate with external load. If the same time of consideration of the process was selected for different loads, the dependence was linear (Fig. 3, straight line 1). However this dependence is inconsistent with certain available experimental data [7]. In studying the influence of the averaging time on the dependence studied, we found that the longer the averaging time, the smaller the angle of inclination of the straight line (Fig. 3, straight line *2*). In this connection, we integrated the filtration equation (Darcy's law [4]) averaged over the layer height with respect to time. Assuming that the pressure at exit from the layer p_0 is equal to zero and the pressure at the point of contact of two zones is constant and equal to p_c, we obtain

$$\frac{T\,(\Gamma - p_c\,[k]/k_1)}{\eta} = \frac{Q}{k_1}\,. \tag{8}$$

Here the difference of the filtration coefficients in highly permeable and low-permeable zones is written in square brackets. The value of p_c must be found from experiments. According to [1], this determination is equivalent to the accurate determination of the constant λ for clay. For the sake of simplicity, we take $p_c \approx \varepsilon\Gamma$, where ε is a small quantity. Now the experimental time is calculated from the formula

$$T = \frac{\mathrm{const}}{\Gamma}\,, \tag{9}$$

where the constant is taken by the researcher. Thus, in actual fact, dependences (8) and (9) correspond to the termination of the process of straining of clay, as a certain amount of the fluid is squeezed. In prescribing the time of consideration of the process, we have obtained the dependence of the shrinkage rate on the load applied to the layer; qualitatively, this dependence has the same form as a typical experimental rheological curve of the rate of shrinkage of clay as a function of the load applied [7] and more accurately describes the process of straining of clay than the Bingham model.

Let us consider the influence of the piezoconductivity coefficients on the dependence of the mean shrinkage rate on the external load. In Fig. 4, it is seen that the shrinkage has become larger for $\chi_1 = 0.7$ and $\chi_2 = 0.07$ than it was for $\chi_1 = 0.3$ and $\chi_2 = 0.03$. Clearly, the higher the permeability of the layer, the larger will be the shrinkage, i.e., this factor, too, is in qualitative agreement with experiment and with the intuitive idea of the process.

Conclusions. The constructed mathematical model of straining of clays yields, at least qualitatively, the same behavior as the real experiment on shrinkage of a clay layer [7]. This enables us to speak of the correspondence of the model to the physics of the occurring processes and to subsequently consider more difficult problems associated with the straining of clay rocks. Clearly, we are dealing with the simplest schematization of the process where the inhomogeneity of actual clay rocks in composition (presence of other mineral phases along with clay minerals) and structure (pore inhomogeneity in size) is allowed for by an abruptly changing effective piezoconductivity coefficient. Investigation of the regularities of straining of a clay layer that more accurately allow for the nonlinear character of the equations for shrinkage in each zone [1] and a layer not entirely saturated with water seems the most promising for further comparison to the existing data on the rheological properties of clay grounds.

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NOTATION

E, Young modulus; k, permeability of clays; l, clay-layer thickness; p, pressure in the liquid phase; p_0, exit pressure in the liquid phase; p_c, pressure in the liquid phase at the point of contact of two shrinkage zones; Q, total amount of water squeezed from the clay; T, total time of the rheological test; t, time; z, vertical axis; Γ, external load; χ, piezoconductivity coefficient; η, water viscosity; λ, elastic constant; θ, shrinkage; ξ, boundary of contact of two shrinkage zones. Subscripts: 0, value of the parameter at exit from the layer; 1, parameter of the first shrinkage zone; 2, parameter of the second shrinkage zone; c, clay.

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